Fractal Model of Nonlinear Hierarchical Complexity: Measuring Transition Dynamics as Fractals of Themselves

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Abstract

Fractal transition theory and measurement enables fine-grained analysis of the most seemingly-chaotic of the developmental transition phases. The explication of the fractal nature of those transition dynamics informs study of learning, decision making, and complex systems in general. A hallmark of the fractal measure is the use of thesis-organized transition measures that are orthogonal to time. Using this method, unpredictable behaviors become "rational" when understood in terms of attractors within developmental processes. An implication for nonlinear science is to transform data otherwise interpreted as incoherent "white noise" into the coherent fractals of the "pink noise" dimension. By integrating Commons *et al's* model of hierarchical complexity (MHC) and this nonlinear model of the fractal transitional orders of hierarchical complexity, a unified mathematical theory of behavioral development will be possible. Such a new formal theory would account for the entire span of behavioral development's equilibrium states and phase transitions, from lowest to highest orders of complexity. The mathematical expressions for the transitional orders of hierarchical complexity must be developed and integrated with the existing MHC.

Keywords: Behavioral development, decision making, developmental transitions, fractal model of nonlinear hierarchical complexity, fractal transition theory, learning, microdevelopment, nonlinear dynamics, phase transitions.

FRACTAL MODEL OF NONLINEAR HIERARCHICAL COMPLEXITY

This paper builds on my previous work (Ross, 2007, 2008; Ross & Commons, 2007) to introduce fractal transition theory and measurement, and on that basis to propose the fractal model of nonlinear hierarchical complexity, and to discuss related issues. One intended contribution is to add to the methodological discourse on transitions in the developmental field, and another is to help forge closer linkages between that field and complexity science. A significant motivation for the paper is to attract expertise to help with the mathematical expression of the transitional orders of complexity presented herein.

To accomplish these purposes, the paper is organized as follows. After situating this contribution within the literature on dynamic transitions, the body of the paper is a critical discussion of pre-fractal and fractal transition measurements used to date in conjunction with the model of hierarchical complexity (see Commons, in this issue). My focus is on the most chaotic phase of transition processes. Within that discussion, I supplement my original description of the fractal nature of that transition and explicitly propose the new model. The closing discussion offers key implications of fractal insights into developmental transitions for behavioral sciences.

Transition Dynamics: Understandings and Methods

Transition refers to one or more movements of a system from one state, phase, or activity to another. As a property of dynamics systems, transitions are widely studied. Transition step sequences between stages of performance situate the model of hierarchical complexity (MHC; Commons, Goodheart, Pekker, et al. 2007; Commons, Trudeau, et al. 1998) in the specialized field of microdevelopment as well as the neo-Piagetian tradition. Its content- and scale-free orders of complexity are fractal by definition (they apply to any actions), properties that situate the MHC in the complexity sciences that study nonlinear systems' phase transitions. Microdevelopment studies include dynamic systems approaches to study humans as developing dynamic systems; the field recognizes transitions in task completion occurring over time scales from minutes to months (Granott & Parziale, 2002). They can also take as long as years for highly complex endeavors (Commons, Ross, & Bresette, 2011; Fischer & Yan, 2002). These and other fields of study pay close attention to transitions over time, because from transitions, new behaviors are constructed and emerge. Predictably, a diverse range of methods are used to study and measure transitions.

Methods in nonlinear sciences use mathematics and graphic data analysis to study phase transitions, and transitions have familiar names like bifurcations, catastrophes, and oscillations to distinguish them; some are more complex dynamics than others. Yet nonlinear methods have not yet recognized vertical increases in complexity, i.e., the developmental implications of many transitions. This is the case despite the ubiquitous discussion of self-organization and emergence, and widespread recognition of chaos preceding self-organization. Goldstein (2002, 2007), however, began making inroads to develop a theory of emergence with increasing levels of complexity, using the concept of self-transcending constructions.

Microdevelopment studies, with their focus on learning and development, may use dynamic systems methods to examine developmental transitions of specific tasks (e.g., infant motor learning, Thelen & Smith, 1994), narrative methods to describe generalizable patterns (e.g., Kuhn, 2002), and ordinal scales or coding schemes (e.g., Basseches & Mascolo, 2010; Gelman, Romo, & Francis, 2002; Parziale, 2002).

Sabelli (1995, n.p.) proposed an interdisciplinary bridge to "interpret non-linear dynamics as a mathematical formulation of dialectic logic" that represented a phase plane in four quadrants of dialectical conditions familiar to Piagetians. The dialectic of thesis, antithesis, synthesis is commonly distilled as *A; B (or not A); A or B; A with B.* Such neo-Piagetian orientations emphasize the repeating patterns of transitions from less to more complex stages. That dialectical tradition is explicit in transition work done by Yan and Fischer (2007), Basseches (1984), Laske (2009), Commons and Richards (2002), Basseches and Mascolo (2010), and Ross (2007, 2008; Ross & Commons, 2007). Yan and Fischer (2007, p. 59) summarized the dynamics in individuals' learning as "change among these patterns [unstable, fluctuating, stable] in a continuous process of self-organization that produces the four types of trends—disorganization, regression, improvement, and stabilization—across sessions" and related at least some aspect of the performances to developmental skill theory levels.

Beyond the dialectical schemes, greater numbers of transition dynamics between stages have been discriminated using narrative and coding methods. Laske's (2009) analyses of interviews incorporated the narrative dialectical transition schemata developed by Basseches (1984), 24 progressions of content-types or "thought forms" that construct dialectical thought. When they reported the addition of transition steps to the model of hierarchical complexity, Commons & Richards (2002) renumbered and added to Piaget's original steps. They based new substeps on premises of choice and signal detection theories, "based on Kuhn and Brannock (1977) and the systematization of that by Commons and Richards

(1984b)" (Commons & Richards, 2002, p. 162). Richards and Commons (1990) had proposed using that same signal detection approach in structured experiments to test for existence of some higher stages.

A prominent commonality across the methods reviewed thus far is the linear time dependence of the measures and thus the analyses. There seems to be only one exception to this norm: the use of theme-organized transition scoring that is orthogonal to time; that is, measurement that is not tethered to the time axis. This means the task scoring remains associated with time but is not organized or measured by linear timing of the dynamics. Both the coactive systems coding process for two-person systems developed by Basseches and Mascolo (2010) and the fractal transitions approach developed by Ross (2007, 2008; Ross & Commons, 2007) enabled fine-grained moment-to-moment analysis of transitions that continue over time, revealing interactional dynamics invisible if chronology drove the analysis. Independently, these researchers found that the true nonlinearity of human behavior is perhaps best revealed by using methods that track the structure and process of changes, regardless of their time stamp. Central to these methods is to identify each thesis-action that emerged and track every associated action until an eventual synthesis completes the transition or the task is abandoned. Basseches and Mascolo's coding scheme is not designed to surface fractal patterns. Thus far, it seems fractal methods for developmental transition analysis appear in only my work cited above. In this paper, I supplement my original description of the fractals in the "chaotic" smash phase transition, and explicitly propose new constructs of transitional orders of hierarchical complexity as part of my fractal transition theory.

Stage Transition Measurements Used with Model of Hierarchical Complexity

Pre-fractal Methods

Hierarchical complexity scoring approaches (Commons, Rodriguez, Miller, Ross, LoCicero, et al, 2007) adopted the expanded transition step scheme proposed by Commons and Richards (2002) to measure transitions from one stage of performance (*n*) to another (*n+1*). That scheme recognized two more kinds of phase-shift dynamics occurring within Piaget's dialectical sequence. The basic ordinal scoring is summarized as follows (adapted from Commons & Richards, 2002, p. 162).

Stage n: Entity operates with temporary equilibrium A (thesis) until transition begins at step 1.
Step 1: A not true - Destabilization
Step 2: B (or not A) - Negation (antithesis)
Step 3: A *or* B - Oscillation (relativism)

Step 4: A *and* B – "Chaos" (attempts at synthesis: smash)

Stage n+1 A *with* B forms new action C - New temporary equilibrium C (synthesis)

Using this scoring system, one can quantify the occurrence and progression of transition processes in task performances at any order of hierarchical complexity. It affords meaningful explanatory power for the "how" of development from one stage to another, in any domain of task actions. It is *coarse-grained* with respect to the step 4 smash dynamics because one cannot use it to score the tasks performed during attempts to reach synthesis.

Therefore, the focus in this paper is on explicating the complex dynamics within step 4's "smash" transition phase. That focus is driven by my interests (a) to enable use of nonlinear methods to measure developmental behavioral dynamics at any scale, and (b) to complete the foundations for a *fractal model of nonlinear hierarchical complexity*. My interest in smash, however, was preceded by others' work. Their early focus on explicating smash used signal detection and choice theory methods for empirical purposes, as mentioned earlier.

> The following steps are not stages in the sense of the general stage model [subsequently named the model of hierarchical complexity]. They are not analytical constructs having the necessary properties of orders and hierarchical complexity and the resulting stages. The steps belong to the realm of empirical science and describe the steps of stage acquisition in an empirically testable manner. (Commons & Richards, 1995, p. 7)

The three substeps were developed to "describe different ways of smashing A and B together, without fully coordinating them…

1. Smash_1 Hits and excess false alarms and misses
2. Smash_2 Hits and excess false alarms
3. Smash_3 Correct rejections and excess misses" (Commons & Richards, 2002, pp. 162-163)

As constructs for experiments with predetermined options and answers, these substeps are *nominal:* they represent *sets of categories* of certain actions, and those actions are nominally described in metaphorical terms. In short, this substep scheme does not support mathematical expression of single actions. It has to be confined to experimental settings that can use it and it has to be excluded from hierarchical complexity transition theory and measurement. I learned the hard way that these substeps are also a mismatch for scoring natural behaviors of an entity (person, group, system, etc.). In all natural behaviors, each person or group generates a unique set of variables to coordinate in the process of eventually arriving at its own synthesis: no one can predict which variables will emerge, or be rejected, or be incorporated in a synthesis until it happens!

For general use in developmental analysis, the straightforward four-step scoring scheme above is sufficient; few of us have a need to delve into micro analysis of the smash step. Yet, in the course of my efforts to measure the nonlinear task dynamics of smash phase transitions, I discovered the behavioral fractals that comprise them (Ross, 2007), and their theoretical and scientific implications (Ross, 2008). That fine-grained fractal transition measurement approach is presented next.

Fractal Method

The significance of presenting the fractal transition measurement is directly connected to the formal, general theory status of the model of hierarchical complexity (MHC; Commons, Goodheart, Pekker, et al. 2007). As a mathematically-based, universal, scale-free behavioral development model, the MHC accounts for the discrete orders of hierarchical complexity of actions. Those orders have enabled us to measure stages of developmental performance for over 30 years. Currently, MHC explains *that* discrete orders of hierarchical complexity are constructed by coordinating lower order actions, but does not yet explain the *how* of those coordinations. Those coordinating actions are discrete for measurement purposes yet occur during continuous living system behaviors. Thus, the MHC does not yet describe *transitional* orders of complexity. Consequently, no formal theory yet accounts for the *continuity* of actions' emergence comprising behavioral development.

To possess "universal, scale-free" properties means the MHC's orders of hierarchical complexity are fractal. Fractal means the repetition of self-similar patterns at different scales. Behavioral scales from the micro-biological to large social systems evidence the orders of hierarchical complexity (see Commons & Ross, 2008). The fractal transition theory is proposed as a universal, scale-free general model as well (Ross, 2008). Its measurement is discussed next.

On the surface, step 4's *A and B* implies only two actions are involved in smash. As the substeps from Commons and Richards suggested, many actions may be performed during this phase. The fractal transition measure accommodates the unpredictable origination of new actions as an entity constructs them nonlinearly in real time. In individuals and groups, these ubiquitous dynamics appear in behavioral processes of learning, reflecting, explaining, problem identification, problem solving, decision making, meaning making, perspective taking, theorizing, and so on.

The transitional orders of hierarchical complexity are ordinal, consistent with the MHC. They are expressed in terms of MHC *primary order's ordinal* paired with the *transition order's ordinal*. For example, a formal operations task performance in the oscillation phase (*A or B*) is indicated as 10-3 for primary order 10, transitional order 3. Every transition begins with some temporary equilibrium (*A*), regardless of when or where a transition occurs. This means transitions nested within transitions are ordered in exactly the same way.

The fractal measurement uses the same math-based transition steps 1 through 3 as presented earlier. If transition is not abandoned after step 3, the smash phase begins with the entity's next action, which is to construct a thesis related to the original task. It is another *A*, temporary equilibrium. What happens next is unpredictable. The new thesis may launch one or more subtasks with varying degrees of nesting within other distinct theses. Regardless of when they emerge in the process, smash subtasks measure as full or partial fractals of the transitional orders of hierarchical complexity (Figure 1).

Once constructed, smash-phase theses are commonly developed as well as temporarily abandoned in a discontinuous fashion. The discontinuity is not a measurement problem. By their nature, the fractal measures transform seemingly random data to reveal their observably coherent order. This is because the fractal transitional orders are orthogonal to time, and transition sequences begin with measureable theses. Figure 2 displays data to illustrate these points.

- The 38 items were sequentially-spoken actions during an 11-minute decision-making (problem-solving) session, and scored using the fractal method. The 38 item numbers are listed along the time axis at the bottom of the figure.
- Items' duration in seconds are shown along the time axis.
- The scores of the sequentially-spoken actions are on the horizontal time axis under their respective item numbers.
- The vertical axis is the relevant range of ascending transitional orders of hierarchical complexity. Actions are measured against them.
- The filled cells communicate two kinds of information.
    - The number in a cell references one of the nine theses constructed during the trial. Thesis numbers range from 1-9.

- The *placement* of the thesis-number in a cell indicates where an action fell on the ordinal scale of complexity. That placement yields the scores given on the horizontal time axis.

If the chronologically sequenced item scores in the bottom row of Figure 2 were plotted on a graph, their erratic discontinuity would be starkly obvious. By contrast, when the data associations with each thesis are maintained, clearly coherent patterns are evident. The patterned data indicate *how* the entity is developing more complex behaviors from moment to moment, and demystify *how* syntheses at higher order task performances are constructed.

Discussion

Living entities are dynamic systems that behave nonlinearly and thus unpredictably, yet always with coherence when we use measures that "let their data speak." When they do, we can "hear" them without distortions of linear time-based assumptions and methods. With measures based on the fractal transitional orders of hierarchical complexity, we can "give voice" to data generated by nonlinear behaviors via the fine-grained analysis of the most complex transition phases.

These unpredictable behaviors are "rational" when understood in terms of *attractors* operating within developmental processes. Each thesis constructed by an entity is an attractor, and the coordination processes return to it at different points in time until it is resolved, or cannot be resolved and is abandoned, or is interrupted and subsequently forgotten or conditions change. This accounts for why there are often-discontinuous actions on a thesis that are nonetheless developmentally coherent. As I previously argued in more detail (Ross, 2008), there is an important implication here for nonlinear science methods: the transformation of data otherwise interpreted as incoherent "white noise" into the coherent fractals of the "pink noise" dimension. Further—and for the first time, I believe— with hierarchical complexity transition measures, complexity science could discriminate if/how systems' phase transitions result in hierarchically greater system complexity. The meaning and implications of many transitions could become more evident and more deeply understood.

Users of the model of hierarchical complexity's previous scoring system now have direction on a scoring method that is internally consistent with the general model. The earlier non-theoretical substeps have theoretically-sound replacements for those who do fine-grained analysis.

I hope one implication of this work for adult development specialists is reinforcement of the understanding that behavior develops in any domain task by task: people are not “at” a stage of development, but rather, day in and day out, they perform tasks at different stages of development and much of the time tasks are in transition phases.

Finally, this work implies that a unified theory of behavioral development is on the horizon: Commons *et al’s* model of hierarchical complexity and this nonlinear model of the fractal transitional orders of hierarchical complexity demand integration. The resulting formal theory would account for the entire span of behavioral development’s equilibrium states (satisfied by the current MHC) as well as the phase transitions (this current proposal) from lowest to highest complexity. The mathematical expressions for the transitional orders of hierarchical complexity must be developed. A large *n* study will be vital so the contributions of these nonlinear developmental measures find their way into sciences of learning, decision making, and complex systems.

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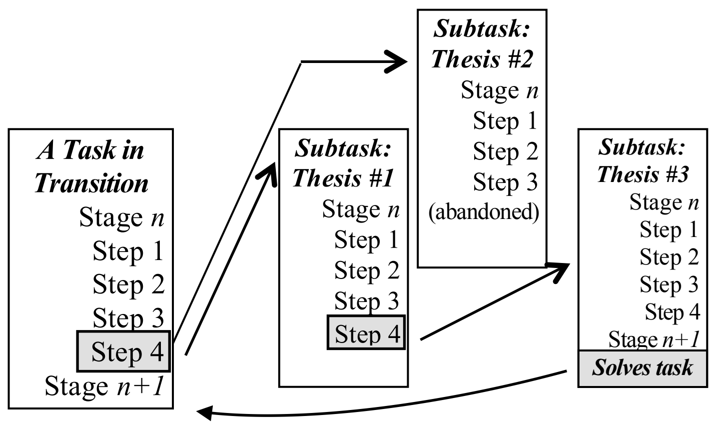

Fig.1 Representation of Smash Phase Fractals of Transitional Orders of Hierarchical Complexity

Orders of Hierarchical Complexity

12-2																	5																					
12-1																5																						
12-0																																						
11-4																											7											
11-3										3																7												
11-2									3																7													9
11-1								3																7													9	
11-0							3					1										6	7													9		
10-4											1								4		2							2							9			
10-3			1			2									4			4		2														9				
10-2		1			2									4																	8		9					
10-1	1			2									4																	8		9						
10-0																													8									
Duration in secs	18	18	22	17	1	14	5	3	12	5	45	5	14	19	5	33	14	27	57	12	13	30	6	19	2	14	17	22	3	4	10	3	1	14	42	35	9	24
Item #	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38
Item Scores	10-1	10-2	10-3	10-1	10-2	10-3	11-0	11-1	11-2	11-3	10-4	11-0	10-1	10-2	10-3	12-1	12-2	10-3	10-4	10-3	10-4	11-0	11-0	11-1	11-2	11-3	11-4	10-4	10-0	10-1	10-2	10-1	10-2	10-3	10-4	11-0	11-1	11-2

Sequential Actions over 11-Minute Trial

Fig. 2 Patterns of Coherence in Data Measured with Fractal Method