

What Are the Relationships between Four Notions of Stage Change

Michael Lamport Commons
Harvard Medical School

Author Note

Correspondence regarding this article should be addressed to Michael Lamport Commons, Harvard Medical School, 234 Huron Avenue, Cambridge, MA 2138. E-mail: commons@tiac.net

Abstract

There have been a number of models for transition between stages, including Piaget's dialectical model, Dawson's use of Rasch score values, and the newest, the systematization resulting from micro-genetic research. This paper discusses four different accounts of stage transition, each delineating how to obtain data on stage transition for each method. It also discusses how the methods might be interrelated. First, the paper elaborates the original Piagetian model by systematizing the transition substeps using choice theory and signal detection. An examination of stage transition included scoring interviews or other participant responses for statements that reflect each of these steps. Secondly, the paper examines micro-developmental approaches. These approaches identify what may be potential subtask and subsubtask actions that may occur during transition to the next stage. Then, the paper describes and illustrates the use of Rasch analysis to quantify the extent to which a participant's performance on an instrument is transitional. This approach might numerically pinpoint where in the transition an individual is but it did not measure the difficulty of the specific task subtask actions (strategies). A method for combining stage scores, subtask action scores, and the sub-subtask action scores was introduced. Finally, the paper presents a methodology for creating tasks and methods of support that directly measure transition. The purpose of this approach was twofold. First was to empirically test for the transition subtask and subsubtask actions extracted originally from the interview process. Second was to figure out how high in transition an action would get with support.

Keywords: stage transition, Rasch analysis, micro-genetic research, micro-developmental approaches, subtask, subsubtask, dialectical theory

What Are the Relationships between Four Notions of Stage Change

There are at least four different versions of stage transition. This paper will discuss what they are and how some of them might be interrelated. Stage transition was early outlined by Piaget in his dialectical theory of stage change. Piaget suggested a dialectical theory of transitional steps. To describe transition, this model elaborates on and systematizes the dialectical strategies described in the Piagetian probabilistic transition model (Flavell, 1963, 1971). This systematization of the substeps was based on choice theory and signal detection (Richards & Commons, 1990).

Transition concepts are represented by a number of different forms of measurement. There are at least four different methods: The conceptions roughly fall into three groups. The first is how steps and strategies are seen in interviews, narratives, writing sample etc. The second is how the steps and strategies are seen from observing how a person works on a task during transition. The tasks are often problems to be solved. The third, is to give instruments that through cuing or some other means, shows what step the person can do with some support.

The four accounts of stage change are: 1. Scoring interviews directly for stage and steps; 2. Micro Development: Designing, Constructing Tasks, Subtask Actions, Subsubtask Actions and Prompts That Measure Transition Directly; 3. Rasch Scaling and Measuring Steps, Subtask Actions, and Subsubtask Actions (Strategies) During Transition; 4. Transition Steps and Substeps.

Scoring interviews directly for stage and steps

In the scoring of interviews and narratives, the interviews and narratives describe task solutions, and the scorer attempts to interpret in the statements how the interviews and narratives reflect the steps in transition. Ross (2008) used this method in her dissertation. The Commons, et al (2005) scoring manual has lots of examples of this. Also see Miller and Lee (2000, June); Miller, Lee, and Commons (2000, June).

After scoring the performances, such as by using a program like Dawson's The Lectical Assessment System (Dawson & Heikkinen, 2009; Stein & Heikkinen, 2008), or scoring by hand, one can perform a Rasch Analysis. Rasch (1980) analysis scales performance and items on the same log linear

line. Transitional performance is shown by the mixtures of performances at different stages. The mixtures range from 0% at the higher stage to 100%. We call 95% at a stage consolidated performance and 0% up to 95% transitional. The advantages of the Rasch analysis are that 1) it reduces measurement variance to a minimum, and (2) thus yields *direct* comparability. The Rasch scores can be translated into stage score. Intermediate stage score show transition. For each part of transition, there are characteristic statements as Dawson has pointed out.

Micro Development: Designing, Constructing Tasks, Subtask Actions, Subsubtask Actions and Prompts That Measure Transition Directly

Informally, macro development refers to changes in behavior and the control of behavior that take place across stages. Micro development refers to changes that take place within or between stages. Micro Development is based on finding what subtask actions, and subsubtask actions (strategies) are used during the transition to the next stage. It is not fractal and they are task sequence dependent. The purpose of the micro development approach is twofold. First is to see what evidence there is for systematic change outside of the interviews process. The second is to systematize the strategies of individuals that one observes in micro-developmental studies. One would see what strategies they used in a number of ways. One could observe how they attack certain stage change examples. Also, one could reinforce correct answers.

By seeing what steps they can do when cued and what strategies they use at each of these steps, one learns what difficulty individuals may overcome and how those individuals overcome them. Note that support in this context is not for a whole stage but in the context of a transitional step between a stage. What is critical is to figure out how high a step they get to, with support. In summary, one could model the step strategy and one could cue it.

Commons-Miller and Commons realized there were subtasks and subsubtasks between the orders of hierarchical complexity. This was clear from Boom et al (in press) work on overlapping waves (See Siegler, 1996) shown with Rasch on stage transition. As inspired by Boom, et al (in press), we differentiate three additional levels of task actions: next higher order tasks, subtask action, and subsubtask action. A *next higher order task* requires that the new task action 1) be defined in terms of two or more next lower order subtask actions, 2) organize them 3) in a non-arbitrary way. A *subtask action* is defined

in terms of 1) only one same order action and 2) another next order or even lower order action. It is not a next order task action because there is only one next lower order task action that the task action operates on. One subtask action is a necessary prerequisite for the next subtask action. A *subsubtask* action is defined in terms of one or more actions two orders down. So they really operate mainly on just one next lower order action. Subsubtask actions may be sufficient but not necessary for the next subsubtask action. The sequence of the actions acquired often depends on the sequence that the teacher provides for the student. There is just one thing that organizes subtasks actions. That is, one subtask action serves as a prerequisite for the next subtask action. The subsubtask actions have a weaker relationship and may be arbitrary organized or one may serve as precursor, and may be only sufficient but not necessary for the next subsuborder task action. The sequence in which these actions are acquired often depends on the sequences that the teacher provides for the students. And it may require more horizontal complexity than the preceding task, as is the case of adding more than two numbers together.

For example, a primary order task is to coordinate preoperational task actions. A preoperational task action coordinates sentential task actions. Coordinating the preoperational order a sentential order task within a primary task action would be subtask.

The metric one might use as output would include the stage scores, subtask action scores, and the subsubtask action scores. These would be $x.y.z$, a three digit decimal that we would consider ordinal. The x = the stage score, the y would be the subtask action score and the z = subsubtask action score. So the first order task, first subtask action, and the first subsubset action would be 1.1.1. If one wants to add stage, it would be (10) 1.1.1 for example.

The three layers of task action may be illustrated with counting and arithmetic tasks. A subtask action within the primary stage is to learn to count. The subsubtasks actions would be the sequence participants go through in moving from preoperational counting of objects in a line to counting objects in a random array. One set of actions come from the preoperational order 6. This requires numbers to be said or indicated for each object in a line array. The problem is that saying numbers from the sentential Oder 5 continues after running out of objects. In primary order 7, one keeps track of what one has counted. This is a prerequisite for the second subtask, which is to learn how to add. Adding is a prerequisite for the third subtask, learning how to multiply. Adding is a subtask action for doing distribution (Long multiplying), but it is not its own next order action. Likewise, multiplying is also a subtask. There are just three things

that organize sub tasks actions. The strongest is a prerequisite, weaker is precursor, and the other is that there is more horizontal complexity.

Preoperational Order 6 Actions Organize Sentential Order 5 Actions

Like all orders in the Model of Hierarchical Complexity (Commons, Trudeau, et al, 1998; and for a earlier and similar model, Fischer, 1980), we characterize Preoperational Order 6 actions by how they are defined, how organisms do the actions, and the end result. Preoperational Order 6 actions are defined in terms of two or more Sentential Order 5 actions. They organize the Sentential Order 5 actions. The organization is non-arbitrary.

For example, organisms form lists of organized set of acts. They make simple deductions that connect simple sequences of actions. Humans telling stories is like sequences of sentences. One of the end results includes that organisms can count random events and objects placed in a row or presented in a sequence, combine numbers, combine simple propositions, and make simple deductions.

Arithmetic at the Preoperational Order

Counting preordered objects is preoperational behavior. Some organisms or preschoolers count. They apply sentential stage ordinal sequences to novel sets of objects placed in a line. One Sentential order 5 task is: 1) Saying numbers in the order they were taught; 2) another sentential order 5 task is pointing to or touching all of a set of objects one by one. The first sentential action is acting out an ordinal sequence by saying the numbers including above the first few. We do not know exactly what the primate representational sequence is, but we do know they have some sort of representation that we will call magnitude (Gallistel, Gelman, 2005). The second sentential stage action is going along the objects in the line one by one. When these two are combined, this is preoperational counting. Specifically, children point to each object in order. Then they apply a number from the number sequence. That number goes up one number in the sequence as they point to a new object. The last count may be called 5, five, cinco, etc. This also indicates the size of the set. This is elementary counting. Because organisms completing tasks at this order relate two sequences together, they may say the sequence of numbers or use number symbols in a sequence, and indicate which object is currently being counted when items are already arranged in a line. Other kinds of sequences may be interrelated as well

Subtasks at the Preoperational Order

Before moving to the primary order, one of the subtasks actions is learning the “tens” labels. Since this must be learned before counting of larger numbers may take place, it is a prerequisite. One of these subtasks actions is learning the “tens” labels. Since this must be learned before counting of larger numbers may take place, it is a prerequisite. Applying the coordinations of number representation to “any number” of objects is required. At first, one counts items, but one does not stop after all the items have been “counted” A subtask at the preoperational order is keeping track of what has been counted. But learning to stop when one runs out of things to count awaits the primary order

Arithmetic at the Primary Order

Counting

At the Primary Order, two or more actions from the Preoperational Order are coordinated. The first subsubtask action is to count disordered objects that are the same. The next subsubtask action is to count disordered objects that are not the same. The last number counted indicates the size of the set. For example, for five objects, the size of the set would be “5”. There are three major subtasks required at Primary Order 7 Counting. The first subtask is true counting. The second subtask is addition and their inverses. The third subtask is multiplication and their inverses.

This first subtask of *true counting* is made possible by the suborder task: 1) Having a way of marking that an object has already been counted by such action as moving it into a separate pile. Primary Order 7 task actions organize Preoperational Order 6 task actions. The first Primary Order 7 subtask actions may organize counts of organized objects from the Preoperational Order 6 tasks and apply them to very large numbers of randomly organized sets of objects. This is done by not only using the counting of objects from the preoperational order, but keeping track of what has been counted, which is also from the preoperational order. Within the “counting” subtask action, there are a number of subsubtask actions. The first subsubtask action is to count disordered objects that are the same. The second is to count disordered objects that are not the same. The third subsubtask in true counting is very large numbers with randomly organized sets of objects. Hence, children count 100's of objects as opposed to 10 to 12. They learn the

subtask actions of addition, subtraction and then multiplication (Van der, Boom, Kroesbergen, & Leseman 2011) and their inverses. This can connect ordinality to cardinality.

Addition

Note that addition is the second subtask and only operates on one action from the primary order, counting. This subtask use of accurate counts is addition/subtraction. This is true adding because they are using symbolic markers to insure that they have counted an item. This can as easily and accurately be done with numbers greater than 10. There are subsubtasks in addition. The first subsubtask action is to learn that the quantity remains constant when two sets are put together. The second subsubtask action is to combine sets. In the first subsubtask action, one counts the second set by continuing the count of the first. In the second subsubtask action, counts of sets can be combined by using the results of the count of the total from subsubtask action one and then continuing the count with the next number. The third subsubtask action is to know what the total of the first set is and what the total of the second set is and then to simply last subsubtask action is to learn the addition table. The third subsubtask action is to add those two numbers together. In literate cultures, the addition facts are learned. The last subsubtask action is to learn the addition table.

Multiplication

Adding is a prerequisite for the third subtask multiplication. There are some subsubtask actions for multiplication. The first subsubtasks for multiplication at the primary order is to see numbers in fixed arrays. An array problem is a problem where there are many rows of items. For example, one could see two rows of three red checkers. This makes it possible to see that groups of numbers have a meaning as opposed to just being "a bunch". The second subsubtask for multiplication at the primary order is to count by multiples of a number. One learns to count by 2's, 3's, 10's etc. The third subsubtask for multiplication at the primary order is to learn the multiplication is repeated addition. This means learning that $2 + 2 + 2$ is the same 3×2 and both answers are 6. This completes learning multiplication is a ways of understanding groups of numbers. The last and fourth subsubtask for multiplication is to memorize multiplication facts. The point to memorizing math facts is to establish automaticity. With automaticity, one recalls facts instantly in lieu of counting on fingers or diagraming the problem Counting strategies.

Rasch Scaling and Measuring Steps, Subtask Actions, and Subsubtask Actions (Strategies) During Transition

Rasch analysis has been used to confirm the order of the hierarchical complexity of stimulus items or tasks (Commons, Goodheart, Pekker, Dawson-Tunik, Cyr, & Rodriguez, 2005). This has been useful in assessing the nature of the items used to measure performance; the possible natural number order of hierarchical complexity of each item, and the corresponding stage of performances on each item. (Mislevy & Wilson, 1996; Spada & McGraw, 1985; Wilson, 1989).

To measure how strategies are distributed during transition, one can take Boom's et al (in press) approach which is to specify the tasks, subtasks and sometimes the subsub tasks by using Rasch Analysis to produce the overlapping waves from the Siegler's (1996) Overlapping Waves Model. This will inform whether or not the subtasks and subsubstaks actions are in the right order and whether they are distinct. Siegler introduced the Overlapping Waves Model as a metaphor to illustrate a typical sequence of increasing and decreasing use of strategies during development. Those strategies may address tasks, subtasks, and subsubtasks. Boom, et al (In press) go beyond metaphor using Item Response Theory (IRT) (Ostini, & Nering, 2006), to analyze such categorical longitudinal data.

Item response theory began with Frederic M. Lord (Lord & Novic, 1968), the Danish mathematician Georg Rasch (1960/1980), and Austrian sociologist Paul Lazarsfeld (1950; 1959). There are a number of people who furthered the progress of IRT (Benjamin Wright, 1990; David Andrich, 2004). IRT provides a framework for evaluating how well individual items on assessments work as well as overall assessments perform. Item response theory focuses on the theory on the item, as opposed to the test-level focus of classical test theory. Item might be multiple choice questions that have incorrect and correct answers or also statements on questionnaires that require participants to rate indicated level of agreement, or patient symptoms scored as present/absent. IRT is based on the idea that the probability of a correct/keyed response to an item is a mathematical function of person and item parameters. The person parameter is called latent trait or ability; it may, for example, represent a person's intelligence or the strength of an attitude. Item parameters include difficulty (location), discrimination (slope or correlation), and pseudoguessing (lower asymptote).

In Boom's analysis, strategy use is scored as an ordinal variable with few categories and

longitudinal development as a vector of such scores. It provides the means to relate the use of such strategies to an underlying developmental dimension. Movement of individuals along this dimension can be modeled by means of Latent Growth Modeling. Latent growth modeling is a statistical technique used in the structural equation modeling (SEM) framework to estimate growth trajectory. It is a longitudinal analysis technique to estimate growth over a period of time. It is widely used in the field of behavioral science, education and social science. Latent Growth Models (Boom, van der Ven, et al, 1958; Rao, 1958; Scher et al 1960; Meredith & Tisak, 1990) represent repeated measures of dependent variables as a function of time and other measures.

Transition Steps and Substeps

Transition steps are somewhat different from transition subtask actions and especially subsubtask actions. For a review of the history, see Commons and Richards (2002) and Ross (2008a). They are more process oriented. They are fractal. Commons and Richards (2002) embellished on Piaget dialectical stage change notion combining Kuhn phases of transition with Piaget's dialectical steps that are also fractal (see Table 1).

So like a dynamical system, an increase in rate of reinforcement may be caused by small perturbations in reinforcement, such as at time b an additional reinforcer being earned. Once perturbed, switching to a new behavior may increase the rate further if it is further along in transition. But, switching is not deterministic, it is probabilistic.

What happens with the effect of reinforcement of switching more often, is that the rate of switching between A and B goes up. For example, the relativistic step, the action of switching is reinforced more often for doing A in certain and B in others, and vice versa. So A occurs in certain situation and gets reinforced, and likewise B can be also is reinforced in certain situations. The world is complicated and probabilistic enough that once in a while a behavior gets paid off. This is an example of melioration, in which the frequency of a response, B , relative to another response, A , increase as the rate of reinforcement for B increases.

The volatility of switching and forming combinations increases dramatically during smash. One cannot predict what combination will occur in first step of smash. One can say it is probabilistically what

they are, but there are a variety of them. The interesting first step of smash before hits emerge is that there is so much variety. This is not a settled down deterministic system. Very small changes of reinforcement during smash, pulls it into the substep in which hits start to predominate. Finding the rate and acceleration of alternations of old-stage and newer-stage actions has never been tried. Finding the proportion of new-stage versus old-stage behavior has been found. A Rasch analysis is a more advanced form of this.

Speculated Rate of Change through the Steps

There is no reinforcement gain in going to step 1, just an experience loss of reinforcement. Therefore, it might take a long time. There is very little gain at step 2 because the alternative does not usually work. There is slightly more at step 3. The flexibility does produce some gains in many cases. The big gains are at step 4, but it is also the most risky. Once into Step 4, progress should be fast because the acceleration of reinforcement as one gets hits is huge.

How Does Stage Change Take Place?

This is a brief summary of stage change interventions. Part of the issue is whether or not it is even possible to create generalizable stage change. There are some dilemmas or a paradox that Plato describes. Plato asserted that one cannot learn anything one does not already know. It is also Fodor's argument against learning. Becker has articulated that the Piagetian notion of reflection or reflective abstraction cannot be true. This is because it means one would already have to know the next stage behavior to reflect upon it. The solution has always been that the transition steps are always driven by gains in reinforcement. Sticking with the previous stage means a loss of potential reinforcement. Sticking which each step also means a loss of potential reinforcement.

Reflections, Metacognition and the Compression of Stages During Recapitulation.

Let us say that reflection is a form of metacognition. At first, reflection on one's performance costs an entire stage. So shadowing what one is doing is one stage higher than just doing the task. As lower stage actions become practiced, they become automatic. When they are automatic, they no longer take up as much "computational space" in M-space terms (Pascual-Leone, 1970; Pascual-Leone, & Goodman, 1979). This would account for part of the compression we see in reflection. Even

metacognitive actions can become automatic (Schrader, 2005, personal communication). The lower stages are integrated into the complexity of the higher stages, becoming one with the higher system as it were. For example, Commons does not think about variables at first when solving algebra problems. Usually, he only thinks of values of variables for just long enough to move on to the variable along which they lie. So compression would be one form, and this rapid recapitulation of the stages would be a second form used in reflection.

But in Schrader (2010) personal communication) and my theory of next stage performance, there is something like chunking (e.g. Gabriel & Mayzner, 1963; Gobet, Lane, et al 2001, Miyapuram, Bapi, Pammi & Kenji, 2006) that transforms lower stage actions into new actions that do not require recapitulation of the lower stage actions. Even though the lower stages are integrated into the complexity of the higher stages, becoming one with the higher system as it were, they probably do not take up more "computational space." Unlike a computational model, however, the systems do not work like a model where the processes remain intact though are processed 'faster' or unconsciously in order to 'make room' for the higher more complex thought processes. They are chunked into units that take less space.

Step 0 and Stage Change

The major problem with the reinforcement theory of stage change lies at step 0. What interventions might work and why? Without intervention and just exposure to the next stage task, the question is how does an organism know that there is more reinforcement ahead that they are not obtaining (Becker, personal communication) without knowing about the next stage is some way? Bereiter (2006) considered 10 relatively neglected resources for the "bootstrapping" of cognitive growth, including chance. Here are some incomplete proposals that capitalize on chance. 1) The organism observes another organism obtaining more reinforcement on the same task. This is incomplete because there is always a first time of observing. Does the organism see the different response to the same task? 2) One possible mechanism is that the present stage action fails to obtain reinforcement period. On such an occasion, Step 0 could begin. 3) It might be such a local failure that it in itself is not overwhelmed by the relatively constant rate of reinforcement. Vaughan and Herrnstein (Herrnstein, & Vaughan, 1980; Vaughan, 1981; Vaughan, & Herrnstein, 1987) showed over and over that the most local rate of reinforcement is what controls behavior. Also, an organism might encounter a new problem for a lot of different reasons. 4) We do see stage change often when the environment abruptly changes. This is the case when gaining cross

cultural experience (Commons, Morse & Galaz, 2006). The place to get stuck is before step 0.

There are three targets of stage change: individuals, groups including cultures, organizations and the like, and species. For human individuals, most of us hope that education will increase stage. Commons, Galaz-Fontes and Morse (2006) found that the average moral stage was concrete stage 8 in non-literate people in Baja California. Contrast this with mean stage of performance in the U.S. to be Formal stage 10 (Commons, 2008). Very preliminary data suggests that the more educational opportunity one provides, the greater the stage. But this is correlational data (Commons, Miller & Kuhn, 1982). Already, the top 40% of the U.S. population goes to college. What needs to be addressed is how to get individualized instruction and motivational techniques used with the rest of the population. But the distribution of stage has been highly resistant to modification above 1 stage increase which is a 2 stage in traditional moral development terms (Schlaefli, Rest, & Thoma, 1985). The degree of benefit for many interventions were studied by Grotzer, Commons, and Davidson (1986).

First, six forms of intervention for individuals are ranked by increasing levels of effectiveness as found by Grotzer, Commons, and Davidson (1986). This was on Inhelder and Piaget (1958) pendulum type problem.

1. Letting person create their own problems. This did the worst with people doing worse after intervention.
2. Practice without feedback on presented problems. There was a small but statistically significant effect.
3. Practice with feedback. This produced no improvement. This might be due to the fact that guessing resulted in such a great deal of punishment from finding out one was wrong half the time
4. 2 levels of support, given direct instruction, walking people through the task. Fischer (Fischer, Hand, & Russell, 1984; Fischer, & Kenny, 1986) reports that this raises the stage of performance by 2.
 - a. Whether or not this generalizes or is permanent is not known.
 - b. 1 level of support, providing examples, providing a model. Fischer (Fischer, Hand, & Russell, 1984; Fischer, & Kenny, 1986) has shown that this raises stage by one. With multiple examples, in related but different task sequences, this produces per meant and

generalized performance change. This is the standard way to teach mathematics. The drawback is that people often quit the activity.

5. Reinforce correct answers. With reinforcement and feedback, they raised the stage of performance from concrete to formal in 75% of 5th and 6th graders. In a follow-up study, such performance did not generalize until the 7th grade.

Why Social and Organizational Stage Goes Up

In the special issue of *World Futures* (2008), there are a number of papers that address this area (e.g. Commons, 2008; Commons, & Goodheart, 2008; Commons, & Ross, 2008; Glock-Grueniech, 2008; Inglis, 2008; Koplowitz, 2008; Robinett, 2008; Ross, 2008b; Ross, & Commons, 2008) Therefore it will not be discussed here.

Discussion

As Sara Ross might say, here we have four systems of viewing stage transition and no real unification. Yes, Rasch analysis may be applied to make sure the sequences of steps, scoring or subtasks and subsubtasks is correct. But that does not fit them into a single system. One way to view this, is that the systems address different aspects of transition and use different methodologies and logics. Scoring is a direct application of the Model of Hierarchical Complexity in combination with dialectical theory, choice theory and fractal theory. Micro development deconfounds micro-genetic studies, by separating task analysis, which is the basis of micro development, from performance. Rasch Analysis and the overlapping waves model are statistical analysis of performance of items.

The way the research is conducted also determines which method is to be used. Direct scoring requires some language or observed action product. Micro development requires observation of task performance and better yet a sequence of tasks at a given order, their subtasks and subsubtasks.

References

- Andrich, D. (2004). Controversy and the Rasch model: A characteristic of incompatible paradigms? *Medical Care, 42*, 1-16.
- Bereiter, C. (2006). Learning and knowledge-building in open-source communities: A social-experiential approach management learning. *Management Learning, 37*(2), 187-214
- Boom J. (in press). A New Model for Strategy Development: Combining Categorical Data Analysis with Growth Modeling. *Journal of Adult Development*.
- Boom, J. (2010). Measuring moral development: Stages as markers along a latent developmental dimension. In W. Koops, D. Brugman, T.J. Ferguson, & A. Sanders (Eds.). *The Development and Structure of Conscience* (pp. 151-168). Hove: Psychology Press.
- Boom, J., Sanne, H. G., Van der Ven, E., Koesbergen H. & Leseman, P. P. M. (in press). Latent growth modeling and item response theory applied to the overlapping waves model.
- Commons, M. L., Goodheart, E. A. (2008). Cultural progress is the result of developmental level of support. *World Futures, 64*, 406-415.
- Commons, M. L., Goodheart, E. A., Pekker, A., Dawson-Tunik, T. L., Cyr, E., Rodriguez, J. A., et al.

- (2005). The relationship between orders of hierarchical complexity and Rasch scaled stage scores: Balance beam, laundry, and counselor-patient task sequences. *Journal of Applied Measurement*, 10(2): 1-11.
- Commons, M. L., & Miller, P. M. (1998). A quantitative behavior analytic theory of development. *Mexican Journal of Experimental Analysis of Behavior*, 24(2), 153-180.
- Commons, M. L., & Richards, F. A. (1984a). A general model of stage theory. In M. L. Commons, F. A. Richards, & C. Armon(Eds.), *Beyond formal operations: Vol. 1. Late adolescent and adult cognitive development* (pp. 120-140). New York: Praeger.
- Commons, M. L., & Richards, F. A. (1984b). Applying the general stage model. In M. L. Commons, F. A. Richards, & C. Armon (Eds.), *Beyond formal operations: Vol. 1. Late adolescent and adult cognitive development* (pp. 141-157). New York: Praeger.
- Commons, M. L., & Richards, F. A. (1995). Behavior analytic approach to dialectics of stage performance and stage change. *Behavioral Development*, 5(2), 7-9.
- Commons, M. L., & Richards, F. A. (2002). Organizing components into combinations: How stage transition works. *Journal of Adult Development*, 9(3), 159-177.
- Commons, M. L., & Ross, S. N. (2008). The hierarchical complexity view of evolution and history. *World Futures*, 64, 399-405.
- Commons, M. L., Trudeau, E. J., Stein, S. A., Richards, F. A., & Krause, S. R. (1998). The existence of developmental stages as shown by the hierarchical complexity of tasks. *Developmental Review*, 8(3), 237-278.
- Flavell, J. H. (1971). Comments on Beilin's the development of physical concepts. In T. Mischel (Ed.), *Cognitive Development and Epistemology* (pp. 121-128). New York: Academic Press.
- Dawson, T., & Heikkinen, K. (2009). Identifying within-level differences in leadership decision making. *Integral Leadership Review*, 9(5).
- Fischer, K. W. (1980). A theory of cognitive development: The control and construction of hierarchies of skills. *Psychological Review*, 87, 477-531.
- Fischer, K. W., Hand, H. H., & Russell, S. (1984). The development of abstractions in adolescents and adulthood. In M. L. Commons, F.A. Richards, & C. Armon (Eds.), *Beyond formal operations: Late adolescent and adult cognitive development* (pp. 43-73). New York: Praeger
- Fischer, K.W., & Kenny, S. L.(1986). Environmental conditions for discontinuities in the development of abstractions. In R. A. Mines & K. S. Kitchener (Eds.), *Adult cognitive development: Methods and models* (pp. 57-75). New York: Praeger.

- Gabriel, R. F., & Mayzner, M. S. (1963). Information "chunking" and short-term retention. *Journal of Psychology: Interdisciplinary and Applied*, 56, 161-164.
- Gobet, F., Lane, P. C. R., Croker, S., Cheng, P. C. H., Jones, G., Oliver, I., & Pine, J. M. (2001). Chunking mechanisms in human learning. *Trends in Cognitive Sciences*, 5, 236-243.
- Grotzer, T. A., McCarthy, K. G., Frome, K.W., & Commons, M. L. (June, 1987). *The process of stage change*. Paper presented at Beyond Formal Operations 3: Positive development during adolescence and adulthood. *The Development of Adolescent and Adult Thought and Perception*, Harvard University, Cambridge.
- Herrnstein, R. J. (1982). Melioration as behavioral dynamism. In M. L. Commons, R. J. Herrnstein, & H. Rachlin (Eds.), *Quantitative analyses of behavior: Vol. 2. Matching and maximizing accounts* (pp. 433-458). Cambridge, MA: Ballinger.
- Herrnstein, R. J., & Vaughan, W., Jr. (1980). Melioration and behavioral allocation. In J. E. R. Staddon (Ed.), *Limits to action: The allocation of individual behavior* (pp. 143-176). New York: Academic Press.
- Commons, M. L., Miller, P. M., Goodheart, E. A., & Danaher-Gilpin, D. (2005). *Hierarchical Complexity Scoring System (HCSS): How to Score Anything*. Unpublished Scoring Manual available from DareAssociation.org
- Lazarsfeld, P. F. (1950). The logical and mathematical foundation of latent structure analysis. In S. A. Stouffer et al. (Eds) *Measurement and Prediction*. Princeton NJ: Princeton University Press
- Lazarsfeld, P. F. (1959). Latent structure analysis. In S. Koch (Ed) *Psychology: A study of a science. Vol 3*. New York: McGraw-Hill.
- Lord, F. M., & Novic, M. R. (1968). *Statistical Theories of Mental Test Scores; with contributions by Allan Birnbaum*. Reading, MA: Addison-Wesley Publishing Co.
- Miller, P. M., & Lee, S. T. (2000, June). *Stages and transitions in child and adult narratives about losses of attachment objects*. Paper presented at the Jean Piaget Society, Montreal, Quebec, Canada.
- Miller, P. M., Lee, S. T., & Commons, M. L. (2000, June). *Scoring for stage and transition of losses in children and adult's lives*. Data presented at Society for research in Adult Development Symposium, Pace University, New York City, New York.
- Mislevy, R. J., & Wilson, M. (1996). Marginal maximum likelihood estimation for a psychometric model of discontinuous development. *Psychometrika*, 61, 41-47.
- Miyapuram, R. S., Bapi, C. V. S. Pammi, A., Kenji D. (2006). *Hierarchical Chunking during Learning of Visuomotor Sequences*. Paper presented at International Joint Conference on Neural Networks,

- Vancouver, BC, Canada. Paper retrieved
http://iitgn.academia.edu/KrishnaPrasadMiyapuram/Papers/272273/Hierarchical_Chunking_during_Learning_of_Visuomotor_Sequences
- Ostini, R., & Nering, M. L. (2006). *Polytomous item response theory models*. Thousand Oaks: Sage.
- Pascual-Leone, J. (1970). A mathematical model for the transition rule in Piaget's developmental states. *Acta Psychologica*, 32, 301-345.
- Pascual-Leone, J., & Goodman, D. (1979). Intelligence and Experience: A neo-Piagetian approach. *Instruct. Sci.*, 8, 301-367
- Rasch, G. (1960/1980). *Probabilistic models for some intelligence and attainment tests*. (Copenhagen, Danish Institute for Educational Research), expanded edition (1980) with foreword and afterward by B.D. Wright. Chicago: The University of Chicago Press.
- Richards, F. A., & Commons, M. L. (1990). Applying signal detection theory to measure subject sensitivity to metacognitive, systematic and lower developmental stages. In M. L. Commons, C. Armon, L. Kohlberg, F. A. Richards, T. A. Grotzer, & J. D. Sinnott (Eds.), *Adult development: Vol 2. Models and methods in the study of adolescent and adult thought* (pp. 175-188). New York: Praeger.
- Ross, S. N. (2008a). Fractal transition steps to fractal stages: The dynamics of evolution, II. *World Futures*, 64, 361-374.
- Ross, S. N. (2008b). Future society functioning at the paradigmatic stage? *World Futures*, 64, 554-562.
- Ross, S. N., & Commons, M. L. (2008). Applying hierarchical complexity to political development. *World Futures*, 64, 480-497.
- Siegler, R. S. (1996). *Emerging minds: The process of change in children's thinking*. New York: Oxford University Press.
- Stein, Z., & Heikkinen, K. (2008). On operationalizing aspects of altitude: An introduction to the Lectical Assessment System for integral researchers. *Journal of Integral Theory and Practice* 3(1), 105-138.
- Van der Ven, S. H. G., Boom, J., Kroesbergen, E. H., & Leseman, P. M. (2011). Relations between the development of strategy selection in learning single-digit multiplication and working memory: A microgenetic study. *Journal of Experimental Child Psychology*, under revision.
- William Vaughan, Jr. (1981). Melioration, matching, and maximization. *Journal of Experimental Analysis of Behavior*, 36(2), 141-149.
- Vaughan, W. Jr., & Herrnstein, R. J. (1987). Stability, Melioration, and Natural Selection. In Green L.,

- and Kagel, J. H. (Eds.), *Advances in behavioral economics* (pp. 185-215). Norwood: Ablex.
- Wilson, M. (1989). Saltus: A psychometric model of discontinuity in cognitive development. *Psychological Bulletin*, *105*, 276-289.
- Wright, B.D. (1992). IRT in the 1990s: Which Models Work Best? *Rasch Measurement Transactions*, *6*(1), 196-200.
- Yan, Z., & Fischer, K. W. (2002). Always Under Construction: Dynamic Variations in Adult Cognitive Microdevelopment, *Human Development*, *45*, 141-160.

Table1

Combining Kuhn Phases of Transition with Piaget's Dialectical Steps

		Piaget			
Step 7.	A		Step 1	Thesis	
Step 1.	A fails A =		New step	Extinction Process	
Step 2	B (or not A)		Step 1	Negation or complementation, Inversion, or alternate thesis	Antithesis
Step 3.	A or B		Step 2	Alternation of thesis and antithesis depending on non-relevant context	Relativism:
Step 4.	Smash A & B together	1 Random hits, false alarms, misses , correct rejections (Smash1)	New step	Random hits, false alarms, and misses, correct rejections (Smash1) Components from A and B are included in a nonsystematic, non-coordinated manner. Incorporates various subsets of all the possible components.	Synthesis
Step 5		2 Hits and excess false alarms		Incorporates subsets producing hits at stage <i>n</i> . Basis for exclusion not sharp (Overgeneralization).	
Step 6		3 Correct Rejections and Hits		Incorporates subsets that produce correct rejections at stage but excess misses. Basis for inclusion not sharp (Undergeneralization).	
Step 7 (Step 0)	A with B		Step 3	New temporary equilibrium	(Correct synthesis and new thesis).